Guidelines on the Evaluation and Expression of Measurement Uncertainty in the Field of Calibration
Basic Principles of Measurement Uncertainty

1. Evaluation of Uncertainty

The uncertainty of the result of a measurement generally consists of several components. They can be grouped into two categories according to the method used to estimate their numerical values:

- **Type A evaluation**
  Calculation of uncertainty is by statistical analysis through repetitive observations.

- **Type B evaluation**
  Calculation of uncertainty is by means other than statistical analysis.

2. Modeling the Measurement Process

- A measurand \( Y \) can be determined from \( N \) inputs quantities \( X_1, X_2, X_3 \ldots X_N \) through a function \( f \):
  \[
  Y = f(X_1, X_2, X_3 \ldots X_N)
  \]

- An estimate of \( Y \), denoted by \( y \), is obtained from \( x_1, x_2, x_3 \ldots x_N \), the estimates of the input quantities \( X_1, X_2, X_3 \ldots X_N \), through the same function \( f \):
  \[
  y = f(x_1, x_2, x_3 \ldots x_N)
  \]

- The uncertainty associated with the estimate \( y \) is obtained by appropriately combining the estimated standard deviation (or standard uncertainty) of each of the input estimate \( x_i \).

3. Type A Evaluation of Standard Uncertainty

- The arithmetic mean for \( n \) independent observations:
  \[
  \bar{q} = \frac{1}{n} \sum_{k=1}^{n} q_k
  \]

- The standard deviation of the mean (estimate the spread of the distribution of the means):
  \[
  s(q) = \sqrt{\frac{1}{n-1} \sum_{k=1}^{n} (q_k - \bar{q})^2}
  \]

- The standard deviation of the mean (estimate the spread of the distribution of the means):
  \[
  s(\bar{q}) = \frac{s(q)}{\sqrt{n}}
  \]

- For an input estimate \( x_i \) determined from \( n \) repeated observations, the Type A standard uncertainty \( u(x_i) \), with degrees of freedom \( v \) is given by:
$u(x_i) = s(x_i)$

- Note: the degree of freedom should always be given when Type A evaluation of an uncertainty component is reported.

4. **Type B Evaluation of Standard Uncertainty**

- Convert a quoted uncertainty to a standard uncertainty from the knowledge of the probability distribution of the uncertainty.

- Commonly used probability distributions:
  - Normal or Gaussian probability distribution
  - Rectangular probability distribution

- Degree of freedom is assumed to be infinite

![Normal Probability Distribution](image)

A normal distribution can be assumed when an uncertainty is quoted with a given confidence level.

For example, a calibration report states that the uncertainty of a voltmeter is ± 0.1 V with a confidence level of 95%. The standard uncertainty of the voltmeter is given by:

$$u(x) = \sigma = \frac{k \cdot \varepsilon}{k} = \frac{0.1}{1.96} = 0.051 \, \text{V}$$

(Note: 95 % level of confidence has a coverage factor of 1.96)
When an uncertainty is given by maximum bound within which all values are equally probable, the rectangular distribution can be assumed. For example, the accuracy of a voltmeter of a specific range is quoted as ± 0.2 V. The standard uncertainty of the voltmeter is given by:

\[ u(x) = \frac{a}{\sqrt{3}} = 0.2 \div \sqrt{3} = 0.115 \text{ V} \]

### 5. Combined Standard Uncertainty

The estimate of a measurand \( Y \) is given by:

\[ Y = f(x_1, x_2, x_3, \ldots, x_N) \]

\[ \Delta y = \frac{\partial f}{\partial x_1} \Delta x_1 + \frac{\partial f}{\partial x_2} \Delta x_2 + \frac{\partial f}{\partial x_3} \Delta x_3 + \cdots + \frac{\partial f}{\partial x_N} \Delta x_N \]

It can be shown that the above equation leads to:

\[ u_c^2(y) = \left( \frac{\partial f}{\partial x_1} \right)^2 u^2(x_1) + \left( \frac{\partial f}{\partial x_2} \right)^2 u^2(x_2) + \left( \frac{\partial f}{\partial x_3} \right)^2 u^2(x_3) + \cdots + \left( \frac{\partial f}{\partial x_N} \right)^2 u^2(x_N) \]

\[ = c_1^2 u^2(x_1) + c_2^2 u^2(x_2) + c_3^2 u^2(x_3) + \cdots + c_N^2 u^2(x_N) \]

The combined standard uncertainty:

\[ u_c(y) = \sqrt{c_1^2 u^2(x_1) + c_2^2 u^2(x_2) + c_3^2 u^2(x_3) + \cdots + c_N^2 u^2(x_N)} \]

where \( c_1, c_2, c_3, \ldots, c_N \) are the sensitivity coefficients

Each component of the combined standard uncertainty could be calculated using either Type A or Type B evaluation method.
6. Coverage Factor of Combined Uncertainty

To determine the coverage factor of combined uncertainty, the effective degree of freedom must be first calculated from the Welch-Satterthwaite formula:

\[
\nu_{\text{eff}} = \sum_{i=1}^{n} \frac{c_i^2 u_i^4(x_i)}{v_i}
\]

Based on the calculated \( \nu_{\text{eff}} \), obtain the \( t \)-factor \( t_p(\nu_{\text{eff}}) \) for the required level of confidence \( p \) from the \( t \)-distribution table.

The coverage factor will be:

\[
k_p = t_p(\nu_{\text{eff}})
\]

7. Expanded Uncertainty

The expanded uncertainty defines an interval about the estimated result \( y \) within which the true value of the measurand \( Y \) is confidently believed to lie. It is given by:

\[
U = k_p \ u_c(y)
\]

The measurand \( Y \) is reported in the following format:

\[
Y = y \pm U
\]

It means that the true value of measurand \( Y \) is confidently believed to fall within the following range:

\[
y - U \leq Y \leq y + U
\]
Example #1: Resistance Measurement

A milliohm meter is used to measure the resistance of a current shunt resistor. At the selected range of the meter for the measurement, the calibration certificate states an uncertainty of ± 0.5 mΩ at 95 % of confidence level. Effects of room temperature and humidity on the measurement are found to be negligible.

\[
\text{Measurement record:} \\
\begin{array}{cccccccccc}
\text{Reading} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
R (\text{mΩ}) & 10.2 & 9.9 & 10.2 & 10.6 & 10.5 & 10.3 & 10.6 & 10.5 & 10.3 & 10.3 \\
\end{array}
\]

1. Measurement Process Model

The measured resistance is given by:

\[
R_x = R_{rdg} + \Delta R_m
\]

where \( R_{rdg} \): resistance reading recorded by the meter

\( \Delta R_m \): meter uncertainty

2. Uncertainty Equation

The combined standard uncertainty is given by:

\[
u_c(R) = \sqrt{u^2(R_{rdg}) + u^2(\Delta R_m)}
\]

Since \( c_1 = \frac{\partial R_x}{\partial R_{rdg}} \sim 1 \) and \( c_2 = \frac{\partial R_x}{\partial (\Delta R_m)} \sim 1 \), the combined standard uncertainty is given by:

\[
u_c(R) = \sqrt{\left(u(R_{rdg})\right)^2 + \left(u(\Delta R_m)\right)^2}
\]

Where

\( u(R_{rdg}) \) is the standard uncertainty due to the repeatability of the meter reading

\( u(\Delta R_m) \) is the standard uncertainty due to the meter calibration
3. Calculation of Uncertainty Components

Type A evaluation:

The best estimate of the measured resistance is given by the arithmetic mean:

\[ \bar{R} = \frac{1}{10} \sum_{k=1}^{10} R_k = \frac{1}{10} \times 103.4 = 10.34 \text{ mΩ} \]

Standard deviation:

\[ s(R) = \sqrt{\frac{1}{10} \sum_{k=1}^{10} (R_k - \bar{R})^2} = 0.217 \text{ mΩ} \]

Standard uncertainty:

\[ u(R_{\text{rms}}) = \frac{s(R)}{\sqrt{n}} = \frac{0.217}{\sqrt{10}} = 0.069 \text{ mΩ} \]

Degree of freedom, \( v = 9 \)

Type B evaluation:

The uncertainty of the calibration is \( \pm 0.5 \text{ mΩ} \) with 95% of confidence level (\( k = 1.96 \)).

\[ u(\Delta R_m) = 0.5 = 0.255 \text{ mΩ} \]

Degree of freedom, \( v = \infty \)

Note: The value of 0.5 mΩ is used as a component for Type B evaluation on the assumption that the drift and stability of the equipment is negligible.

4. Uncertainty Budget Table

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Type</th>
<th>Uncertainty Value (mΩ)</th>
<th>Probability Distribution</th>
<th>( K )</th>
<th>( u_i ) (mΩ)</th>
<th>( c_i )</th>
<th>( c_i \times u_i )</th>
<th>( V_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repeatability ( u(R_{\text{rms}}) )</td>
<td>A</td>
<td>0.069</td>
<td>-</td>
<td>-</td>
<td>0.069</td>
<td>1</td>
<td>0.069</td>
<td>9</td>
</tr>
<tr>
<td>Meter Calibration ( u(\Delta R_m) )</td>
<td>B</td>
<td>0.5</td>
<td>Normal</td>
<td>1.96</td>
<td>0.255</td>
<td>1</td>
<td>0.255</td>
<td>\infty</td>
</tr>
</tbody>
</table>

5. Combined Standard Uncertainty

\[ U_c (R) = \sqrt{0.069^2 + 0.255^2} = 0.264 \text{ mΩ} \]
6. **Effective Degrees of Freedom**

\[ V_{\text{eff}} = \frac{0.264^4}{0.069^4 + 0.255^4} \]

\[ V_{\text{eff}} - 1928 \]

\[ V_{\text{eff}} - \infty \]

7. **Expanded Uncertainty**

For \( v_{\text{eff}} = \infty \), the coverage factor of the combined standard uncertainty \((k_p)\) is equal to 1.96 at 95 % level of confidence.

\[ U = k_p \times U_c = 1.96 \times 0.264 = 0.517 \text{ m\Omega} \]

8. **Reporting of Result**

\[ R = 10.34 \pm 0.517 \text{ m\Omega} \]

The measured resistance of the current shunt resistor is 10.34 m\Omega. The expanded uncertainty is \( \pm 0.517 \text{ m\Omega} \) with a coverage factor of 1.96, assuming a normal distribution at a level of confidence of 95 %.
Example #2: Temperature Measurement

A digital thermometer with a Type J thermocouple is used to measure the temperature inside a temperature chamber. The temperature controller of the chamber is set at 500°C.

Digital thermometer specification:
- Accuracy = ± 0.3°C.

Thermocouple specifications:
- Temperature correction for the thermocouple at 500°C is 0.5 ± 1.0 °C at 95% confidence level
- Deviation due to immersion = ± 0.2 °C
- Deviation due to drift = ± 0.3 °C

Measurement record:

<table>
<thead>
<tr>
<th>S/N</th>
<th>T (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500.1</td>
</tr>
<tr>
<td>2</td>
<td>500.0</td>
</tr>
<tr>
<td>3</td>
<td>500.1</td>
</tr>
<tr>
<td>4</td>
<td>499.9</td>
</tr>
<tr>
<td>5</td>
<td>499.9</td>
</tr>
<tr>
<td>6</td>
<td>500.0</td>
</tr>
<tr>
<td>7</td>
<td>50.1</td>
</tr>
<tr>
<td>8</td>
<td>500.2</td>
</tr>
<tr>
<td>9</td>
<td>500.0</td>
</tr>
<tr>
<td>10</td>
<td>499.9</td>
</tr>
</tbody>
</table>

1. Measurement Process Model

The measured temperature is given by:

\[ t_x = t_{rdg} + \Delta t_m + \Delta t_c + \Delta t_{imm} + \Delta t_{drift} \]

Where
- \( t_{rdg} \) is the temperature reading recorded by the digital thermometer
- \( \Delta t_m \) is the accuracy of digital thermometer
- \( \Delta t_c \) is the temperature correction of the thermocouple
- \( \Delta t_{imm} \) is the deviation due to immersion of the thermocouple
- \( \Delta t_{drift} \) is the deviation due to drift of the thermocouple
2. Uncertainty Equation

\[ u_c(T) = \sqrt{u^2(T_{rel}) + u^2(\Delta t_{w}) + u^2(\Delta t_{c}) + u^2(\Delta t_{imm}) + u^2(\Delta t_{corr})} \]

All the sensitivity coefficients are equal to unity.

3. Calculation of Uncertainty Components

Type A evaluation:
The best estimate of the measured temperature is given by the arithmetic mean:

\[ \overline{T} = \frac{1}{10} \sum_{k=1}^{10} T_k = 500.02^\circ C \]

Standard deviation:

\[ s(T) = \sqrt{\frac{1}{10-1} \sum_{k=1}^{10} (T_k - \overline{T})^2} = 0.103^\circ C \]

Standard uncertainty:

\[ u(T_{rel}) = \frac{s(T)}{\sqrt{n}} = \frac{0.103}{\sqrt{10}} = 0.033^\circ C \]

Degree of freedom, \( V = 9 \)

Type B evaluation:
The accuracy of the digital thermometer = \( \pm 0.6^\circ C \). Assume rectangular distribution, the standard uncertainty of the digital thermometer meter:

\[ u(\Delta t_{dev}) = \frac{0.3}{\sqrt{3}} = 0.173^\circ C \]

Degree of freedom, \( = \infty \)

The uncertainty of the temperature correction of the thermocouple = \( \pm 1.0^\circ C \) at 95% confidence level (\( k = 1.96 \)). The standard uncertainty due to temperature correction:

\[ u(\Delta t_{c}) = \frac{1.0}{1.96} = 0.510^\circ C \]

Degree of freedom, \( = \infty \)

The uncertainty of the thermocouple due to immersion = \( \pm 0.2^\circ C \). Assume rectangular distribution, the standard uncertainty due to immersion:

\[ u(\Delta t_{imm}) = \frac{0.2}{\sqrt{3}} = 0.115^\circ C \]

Degree of freedom, \( = \infty \)
The uncertainty of the thermocouple due to drift = ± 0.3 °C. Assume rectangular distribution, the standard uncertainty due to drift:

\[ u(\Delta t_{\text{drift}}) = \sqrt{\frac{0.3}{3}} = 0.173 \text{ °C} \]

4. Uncertainty Budget Table

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Type</th>
<th>Uncertainty Value (°C)</th>
<th>Probability Distribution</th>
<th>K</th>
<th>(U_i(°C))</th>
<th>(C_i)</th>
<th>(C_i x U_i)</th>
<th>(V_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repeatability (u(t_{\text{rdg}}))</td>
<td>A</td>
<td>0.033</td>
<td>-</td>
<td>-</td>
<td>0.033</td>
<td>1</td>
<td>0.033</td>
<td>9</td>
</tr>
<tr>
<td>Digital Thermometer (u(\Delta t_{\text{m}}))</td>
<td>B</td>
<td>0.3</td>
<td>Rectangular</td>
<td>1.732</td>
<td>0.173</td>
<td>1</td>
<td>0.173</td>
<td>∞</td>
</tr>
<tr>
<td>Temperature correction (u(\Delta t_{\text{tc}}))</td>
<td>B</td>
<td>1.0</td>
<td>Normal</td>
<td>1.96</td>
<td>0.510</td>
<td>1</td>
<td>0.510</td>
<td>∞</td>
</tr>
<tr>
<td>Immersion (u(\Delta t_{\text{imm}}))</td>
<td>B</td>
<td>0.2</td>
<td>Rectangular</td>
<td>1.732</td>
<td>0.115</td>
<td>1</td>
<td>0.115</td>
<td>∞</td>
</tr>
<tr>
<td>Drift (u(\Delta t_{\text{drift}}))</td>
<td>B</td>
<td>0.3</td>
<td>Rectangular</td>
<td>1.732</td>
<td>0.173</td>
<td>1</td>
<td>0.173</td>
<td>∞</td>
</tr>
</tbody>
</table>

5. Combined Standard Uncertainty

\[ U_c(t_x) = \sqrt{0.033^2 + 0.510^2 + 0.115^2 + 0.173^2 + 0.58^2} \] °C

6. Effective degrees of freedom

\[ V_{\text{eff}} = \sqrt{\frac{0.33^4}{9} + \frac{0.173^4}{\infty} + \frac{0.510^4}{\infty} + \frac{0.115^4}{\infty} + \frac{0.173^4}{\infty}} \]

\[ = \frac{858,813}{9} \]

\[ = \infty \]
EX #3 DC VOLTMETER CALIBRATION

1.0 INTRODUCTION
The manufacturer's calibration procedure of a multimeter required that a DC voltage of 10V from a multifunction calibrator is applied to the multimeter to verify its DC 20V range accuracy. The specifications of the multifunction calibrator and digital multimeter are as follows:

![Connection Diagram]

Figure 1 Connection Diagram

1.1 Multifunction Calibrator Specifications
The calibrator is calibrated every 90 days. The last calibration was carried out at a 23°C ± 1°C environment and the calibrator was verified to be within specifications.

DC Voltage Specifications

<table>
<thead>
<tr>
<th>Range</th>
<th>Resolution</th>
<th>Uncertainty at 99% level of confidence ± 5°C from calibration temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>24 Hours</td>
</tr>
<tr>
<td>20 V</td>
<td>1 μV</td>
<td>4 ± 3</td>
</tr>
</tbody>
</table>

1.2 Digital Multimeter Specifications

<table>
<thead>
<tr>
<th>DC Voltage range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>20 V</td>
</tr>
</tbody>
</table>

Note: <sup>1</sup> Traditionally the term accuracy is used
It was given in the service manual that a DC voltage of 10V is used to verify the multimeter's DC 20V range. The verification specification for the multimeter DC 20V range is:

**Verification specification**

<table>
<thead>
<tr>
<th>Range</th>
<th>Input Voltage</th>
<th>Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 V</td>
<td>10 V</td>
<td>± 0.0007 V</td>
</tr>
</tbody>
</table>

**Measurement Record**

Temperature: 23°C ± 2°C

Only one reading was taken as the indicated voltage remained unchanged. Errors due to loading effect and connection are negligible.

<table>
<thead>
<tr>
<th>Applied Voltage</th>
<th>Indicated Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.000000 V</td>
<td>10.0001 V</td>
</tr>
</tbody>
</table>

### 2.0 MATHEMATICAL MODEL

The multimeter was connected directly to the calibrator. The model of the process is a function of voltages:

\[ V_{DM} = f(V) = V_{STD} + \Delta V_{DM} \]  \hspace{1cm} (1)

where:
- \( V_{DM} \) is the multimeter indicated voltage
- \( V_{STD} \) is the applied voltage from the calibrator
- \( \Delta V_{DM} \) is the error of the multimeter

### 3.0 UNCERTAINTY EQUATION

The input quantities are uncorrelated. The combined standard uncertainty \( u_c(V) \) is:

\[ u_c^2(V) = \sum_{k=1}^{N} \left( \frac{\partial f}{\partial V_k} \right)^2 u_i^2(V_k) \] \hspace{1cm} (2)

The voltage from the calibrator is directly applied to the multimeter. Since errors due to loading effect and connection were found negligible, the components of the total measurement uncertainty consist of:

- \( u_i(V) \) : the calibrator's applied voltage uncertainty, and
- \( u_r(V) \) : the multimeter's random effect uncertainty.

The combined standard uncertainty becomes:
\[ u_{2}^{2}(V) = \left( \frac{\partial V_{\text{DMM}}}{\partial V_{\text{STD}}} \right)^{2} u_{1}^{2}(V) + \left( \frac{\partial V_{\text{DMM}}}{\partial \Delta V_{\text{DMM}}} \right)^{2} u_{2}^{2}(V) \]

\[ = (c_{1})^{2} u_{1}^{2}(V) + (c_{2})^{2} u_{2}^{2}(V) \]

The sensitivity coefficients are:

\[ c_{1} = \frac{\partial V_{\text{DMM}}}{\partial V_{\text{STD}}} = 1 \] (6)

\[ c_{2} = \frac{\partial V_{\text{DMM}}}{\partial \Delta V_{\text{DMM}}} = 1 \] (7)

The combined standard uncertainty becomes:

\[ u_{c}^{2}(V) = u_{1}^{2}(V) + u_{2}^{2}(V) \]

\[ u_{c}(V) = \sqrt{u_{1}^{2}(V) + u_{2}^{2}(V)} \] (8)

4.1 TYPE A EVALUATION

This is a case where the reference standard's accuracy is much better than the device under test's. The multimeter's reading may remain unchanged, or sometimes the multimeter has a ±1 count flickering due to the multimeter's digitizing process. In this case, the Type A evaluation of standard uncertainty can be assumed negligible and the repeatability uncertainty can be treated as Type B uncertainty using the resolution error of the multimeter.

4.2 TYPE B EVALUATION

4.2.1 Calibrator

From the calibrator's specification, the uncertainty of the applied voltage is:

\[ \sigma_{1} = 5 \text{ ppm of output} + 4 \mu V \]

\[ = 5 \times 10^{-6} \times 10 V + 4 \times 10^{-6} V \]

\[ = 54 \times 10^{-6}(V) \] (7)

Given level of confidence is 99%. Assume normal distribution, coverage factor k=2.58, the standard uncertainty of applied voltage is:
\[ u_1(V) = \frac{\sigma_1}{2.55} = \frac{5.4 \times 10^{-8}}{2.55} = 2.09 \times 10^{-8} \text{ (V)} \] \[ \text{Degrees of freedom:} \quad v_1 = \infty \]

4.2.2 Multimeter:

From multimeter's specifications, the 20V range resolution is 100μV (i.e. 1 count). Since the reading was unchanged, assumed the limit is half a count.

\[ \sigma_2 = \frac{100\mu V}{2} = 50 \times 10^{-8} \text{ (V)} \]

Assumed rectangular distribution, the standard uncertainty due to the resolution uncertainty of the multimeter is:

\[ u_2(V) = \frac{\sigma_2}{\sqrt{3}} = \frac{50 \times 10^{-8}}{\sqrt{3}} = 28.8 \times 10^{-8} \text{ (V)} \] \[ \text{Degrees of freedom:} \quad v_2 = \infty \]

5.0 **COMBINED STANDARD UNCERTAINTY \( u_c(V) \)**

The combined standard uncertainty of the indicated voltage at the multimeter is:

\[ u_c(V) = \sqrt{u_1^2(V) + u_2^2(V)} \]

\[ = \sqrt{(2.09 \times 10^{-8})^2 + (28.8 \times 10^{-8})^2} \]

\[ = 35.7 \times 10^{-8} \text{ (V)} \]

6.0 **EFFECTIVE DEGREES OF FREEDOM \( v_{\text{eff}} \)**

Since \( v_1 \) and \( v_2 \) are infinite, Effective degrees of freedom \( v_{\text{eff}} = \infty \)
It was given in the service manual that a DC voltage of 10V is used to verify the multimeter's DC 20V range. The verification specification for the multimeter DC 20V range is:

**Verification specification**

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</thead>
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</tr>
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**Measurement Record**

Temperature: 23°C ± 2°C

Only one reading was taken as the indicated voltage remained unchanged. Errors due to loading effect and connection are negligible.

<table>
<thead>
<tr>
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</thead>
<tbody>
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<td>10.0001 V</td>
</tr>
</tbody>
</table>

### 2.0 MATHEMATICAL MODEL

The multimeter was connected directly to the calibrator. The model of the process is a function of voltages:

\[
V_{\text{CAL}} = f(V) = V_{\text{STD}} + \Delta V_{\text{CAL}}
\]  

where:

- \( V_{\text{CAL}} \) is the multimeter indicated voltage
- \( V_{\text{STD}} \) is the applied voltage from the calibrator
- \( \Delta V_{\text{CAL}} \) is the error of the multimeter

### 3.0 UNCERTAINTY EQUATION

The input quantities are uncorrelated. The combined standard uncertainty \( u_c(V) \) is:

\[
u_c^2(V) = \sum_{k=1}^{N} \left( \frac{\partial f}{\partial V_k} \right)^2 u_k^2(V_k)
\]

The voltage from the calibrator is directly applied to the multimeter. Since errors due to loading effect and connection were found negligible, the components of the total measurement uncertainty consist of:

- \( u_c(V) \): the calibrator's applied voltage uncertainty, and
- \( u_r(V) \): the multimeter's random effect uncertainty.

The combined standard uncertainty becomes:
\[ u_{\text{eff}}^2 = \frac{u_{1}^2(V)}{v_1} + \frac{u_{2}^2(V)}{v_2} = \frac{\left(3.57 \times 10^{-3}\right)^4}{\left(1 \times 20.9 \times 10^{-6}\right)^4 + \left(1 \times 28.9 \times 10^{-6}\right)^4} \]

\[ u_{\text{eff}}^2 = \ldots \]

7.0 EXPANDED UNCERTAINTY U

From Student's "t" table, for degrees of freedom \(v_{\text{eff}} = \infty\), at 95% level of confidence, the "t" factor is

\[ t_{95}(v_{\text{eff}} = \infty) = 1.96 \]  \( (16) \)

The coverage factor is therefore \( k = 1.96 \)

The expanded uncertainty is:

\[ U = k \cdot u_{\text{eff}}(V) \]
\[ = 1.96 \times 35.7 \times 10^{-6} \]
\[ = 70 \mu V \]  \( (16) \)

8.0 UNCERTAINTY BUDGET

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Type</th>
<th>( u_i )</th>
<th>Uncertainty Value</th>
<th>Sensitivity Coefficient</th>
<th>Probability Distribution</th>
<th>Coverage Factor</th>
<th>Standard Uncertainty (V)</th>
<th>Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrator</td>
<td>B</td>
<td>( u_i(V) )</td>
<td>54 \times 10^{-6}</td>
<td>1</td>
<td>Normal</td>
<td>2.58</td>
<td>20.9 \times 10^{-6}</td>
<td>\infty</td>
</tr>
<tr>
<td>Resolution</td>
<td>B</td>
<td>( u_i(V) )</td>
<td>50 \times 10^{-6} V</td>
<td>1</td>
<td>Rectangular</td>
<td>\sqrt{3}</td>
<td>28.9 \times 10^{-6}</td>
<td>\infty</td>
</tr>
<tr>
<td>Multimeter</td>
<td>Combined</td>
<td>( u_i(V) )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>35.7 \times 10^{-6}</td>
<td>\infty</td>
</tr>
<tr>
<td>Multimeter</td>
<td>Expanded</td>
<td>U</td>
<td>70.0 \times 10^{-6}V</td>
<td>-</td>
<td>t</td>
<td>1.96</td>
<td>-</td>
<td>\infty</td>
</tr>
</tbody>
</table>
9.9 REPORTING OF RESULT

\[ V_{\text{CMV}} = 10.0001 \pm 70 \mu V \]  \hspace{1cm} (17)

The expanded uncertainty is \( U = k u_c(y) \), with \( U \) determined from a combined uncertainty \( u_c \) and a coverage factor \( k=1.96 \). Since it can be assumed that the possible estimated values are approximately normally distributed with approximate standard deviation \( u_c \), the unknown value can be asserted to lie in the interval defined by \( U \) with a level of confidence of approximately 95%.

Taking into consideration the measurement uncertainty of \( \pm 70 \mu V \), the multimeter compliances with the verification specification limits requirement.
References:


## ISSUE AND AMENDMENT RECORD

<table>
<thead>
<tr>
<th>Title</th>
<th>Issue</th>
<th>Date</th>
<th>Amendment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluation and Expression of Measurement Uncertainty in the Field of Calibration</td>
<td>01</td>
<td>January 2015</td>
<td>Initial Issue</td>
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